Input-Driven Stack Automata

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- Extensions/generalizations: multiple stacks, graph automata, height-deterministic PDA, ...

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- Non-erasing variants: no pop-moves are allowed.
- Deterministic and nondeterministic variants.
- One-way and two-way variants.
- Decidability for one-way stack automata: emptiness is decidable for nondeterministic machines, equivalence with regular sets is decidable for deterministic machines.

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We consider also weak variants of input-driven stack automata. Here, the moves inside the pushdown store don't have to be input-driven.

Comparing weak and strong variants

Lemma

$L_1 = \{ a^n b a^{n-1} c \mid n \geq 1 \}$ belongs to $\mathscr{L}(1\text{wNEDVSA})$, but is not accepted by any 1sDVSA.

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It can be shown by pumping arguments that L_1 is not accepted by any 1sDVSA.

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- Every VPDA can be simulated by some equivalent $\mathrm{1sDVSA}$.

Comparing erasing and non-erasing variants

Lemma

$L_3 = \{ a^{n+m} b^m w \hat{w}^R b^n \mid m, n \geq 1, w \in \{0,1\}^* \}$ belongs to $\mathscr{L}(\mathrm{1sDVSA})$, but is not accepted by any $\mathrm{1wNEDVSA}$.

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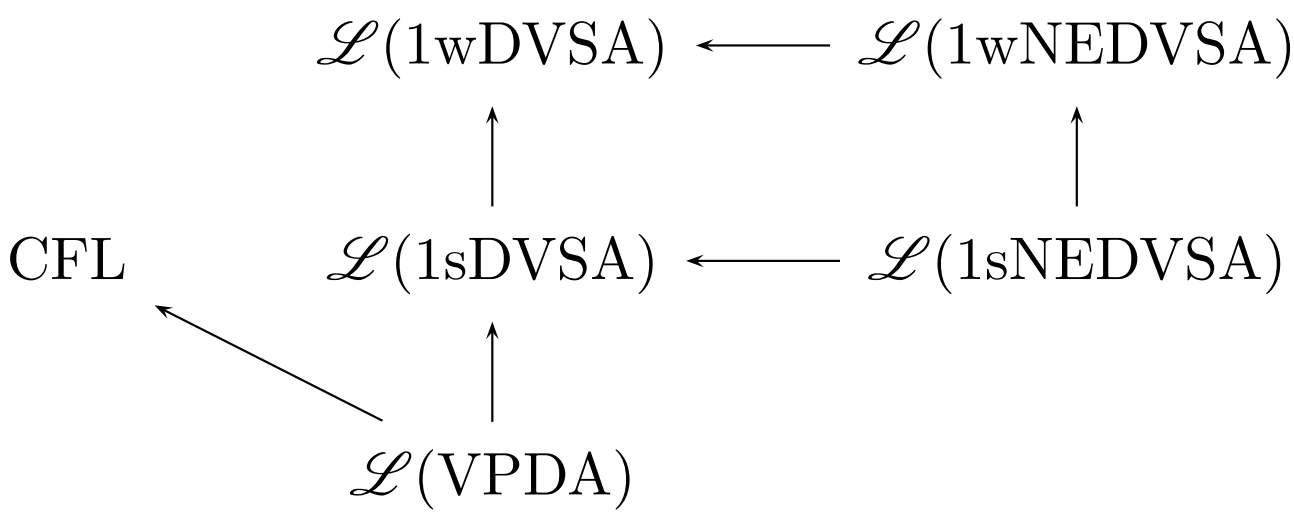
- L_3 is a context-free language where erasing is essential.

Summary for deterministic machines



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Lemma

L_4 belongs to $\mathscr{L}(\text{1sNENVSA})$, but is not accepted by any 1wDVSA.

Comparing deterministic and nondeterministic variants

It is possible to separate all considered deterministic classes from there nondeterministic counterparts.

Corollary

- $\mathscr{L}(\mathrm{1sNEDVSA}) \subset \mathscr{L}(\mathrm{1sNENVSA})$.
- $\mathscr{L}(\mathrm{1sDVSA}) \subset \mathscr{L}(\mathrm{1sNVSA})$.
- $\mathscr{L}(\mathrm{1wNEDVSA}) \subset \mathscr{L}(\mathrm{1wNENVSA})$.
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This is a large difference to the situation for VPDA.

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Theorem

Each of the language families $\mathscr{L}(\mathrm{1sNENVSA})$, $\mathscr{L}(\mathrm{1sNVSA})$, $\mathscr{L}(\mathrm{1wNENVSA})$, and $\mathscr{L}(\mathrm{1wNVSA})$ has an NP-complete fixed membership problem.

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- ➜ Investigate other decidability problems.

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- ➜ Erasing variants are more powerful than non-erasing variants.
- ➜ Relaxing the motion inside the pushdown store is more powerful than input-driven motion.
- ➜ Fixed membership problem remains NP-complete.

Next steps

- ➜ Investigate closure properties of VSA.
- ➜ Investigate the time complexity of the emptiness problem.
- ➜ Investigate other decidability problems.
- ➜ Investigate the relations between nondeterministic variants.